

HW Pre-Calculus 11 Section 6.3 Adding and Subtracting Rational Expressions

1. Unscramble the words to complete the following sentence:

When adding or subtracting two fractions, they must have a "moncmo tmaenodnior"

COMMON DENOMINATOR

2. Given the three rational expressions, what is the common denominator?
- $\frac{1}{2x}, \frac{1}{x+2}, \frac{1}{x-2}$

$$\frac{1}{2x(x+2)(x-2)}$$

3. Add or Subtract each of the following rational expressions:

a) $\frac{x}{2} - \frac{8}{3x}$ $\frac{3x^2}{6x} - \frac{16}{6x}$ $= \frac{3x^2 - 16}{6x}$	b) $\frac{5}{x} - \frac{2}{x+1}$ $\frac{5x+5}{x(x+1)} - \frac{2x}{x(x+1)}$ $= \frac{3x+5}{x(x+1)}$	c) $\frac{3}{x+1} + \frac{2}{x-1}$ $\frac{3(x-1)}{(x+1)(x-1)} + \frac{2(x+1)}{(x+1)(x-1)}$ $\frac{3x-3+2x+2}{(x+1)(x-1)} = \frac{5x-1}{(x+1)(x-1)}$
d) $\frac{2}{x+5} + \frac{3}{x^2+5x}$ $\frac{2x}{(x+5)(x)} + \frac{3}{(x)(x+5)}$ $\frac{2x+3}{(x)(x+5)}$	e) $\frac{5x}{2} + \frac{3}{x} - \frac{4x}{3}$ $\frac{15x^2}{6x} + \frac{18}{6x} - \frac{8x^2}{6x}$ $\frac{7x^2-18}{6x}$	f) $\frac{4}{x+8} - \frac{3}{x} + \frac{2}{3}$ $\frac{4(3x)}{(x+8)3x} - \frac{3(3)(x+8)}{3x(x+8)} + \frac{2x(x+8)}{3x(x+8)}$ $\frac{12x - 9x - 72 + 2x^2 + 16x}{3x(x+8)}$ $= \frac{(2x^2 + 19x - 72)}{3x(x+8)}$
g) $\frac{5x}{3x+9} - \frac{4x}{2x+6}$ $\frac{5x}{3(x+3)} - \frac{4x}{2(x+3)}$ $\frac{5x}{3(x+3)} - \frac{2x(3)}{(x+3)(3)}$ $= \frac{-x}{3(x+3)}$	h) $\frac{9}{x+2} + \frac{7}{x^2-4}$ $\frac{9(x-2)}{(x+2)(x-2)} + \frac{7}{(x+2)(x-2)}$ $= \frac{9x-18+7}{(x+2)(x-2)}$ $= \frac{9x-11}{(x+2)(x-2)}$	i) $\frac{5}{x-1} + \frac{4x}{1-x} + \frac{x+3}{x-1}$ $\frac{5}{x-1} - \frac{4x}{x-1} + \frac{x+3}{x-1}$ $= \frac{8-3x}{x-1}$

4. Factor and simplify. Then state all the NPV's:

<p>a) $\frac{4x^2 - 20x}{x^2 + 2x - 35} + \frac{3x - 6}{x^2 - 12x + 20}$</p> <p>$\frac{4x(x-5) + 3(x-2)}{(x+7)(x-5)(x-2)(x-10)}$</p> <p>$\frac{4x(x-10) + 3(x+7)}{(x+7)(x-10)}$</p> <p>$= \frac{16x^2 - 37x + 21}{(x+7)(x-10)}$</p> <p>NPV's: $x \neq -7, 5, 2, 10$</p>	<p>b) $\frac{2x - 6}{x^2 - 5x + 6} - \frac{3x - 12}{x^2 - x - 12}$</p> <p>$\frac{2(x-3) - 3(x-4)}{(x-3)(x+2)(x-4)(x+3)}$</p> <p>$\frac{2(x+3) - 3(x+2)}{(x+2)(x+3)}$</p> <p>$= \frac{2x + 6 - 3x - 6}{(x+2)(x+3)}$</p> <p>$= \frac{-x}{(x+2)(x+3)}$</p> <p>NPV's: $x \neq 3, -2, 4, -3$</p>	<p>c) $\frac{2x}{3x^2 - 11x + 6} - \frac{3x - 12}{3x^2 - 14x + 8}$</p> <p>$\frac{2x}{(3x-2)(x-3)} - \frac{3(x-4)}{(3x-2)(x-4)}$</p> <p>$\frac{2x - 3(x-3)}{(3x-2)(x-3)}$</p> <p>$= \frac{2x - 3x + 9}{(3x-2)(x-3)}$</p> <p>$= \frac{9-x}{(3x-2)(x-3)}$</p> <p>NPV's: $x \neq \frac{2}{3}, 3, 4$</p>
<p>d) $\frac{2x}{3-x} - \frac{3x}{x+3} + \frac{2}{x^2-9}$</p> <p>$\frac{-2x}{x-3} - \frac{3x}{x+3} + \frac{2}{(x+3)(x-3)}$</p> <p>$\frac{-2x(x+3) - 3x(x-3) + 2}{(x-3)(x+3)}$</p> <p>$= \frac{-2x^2 - 6x - 3x^2 + 9x + 2}{(x-3)(x+3)}$</p> <p>$= \frac{-5x^2 + 3x + 2}{(x-3)(x+3)}$</p> <p>NPV's: $x \neq 3, -3$</p>	<p>e) $\frac{w-6}{w^2-12w+36} + \frac{7w+56}{w^2+11w+24}$</p> <p>$\frac{(w-6)}{(w-6)(w-6)} + \frac{7(w+8)}{(w+3)(w+8)}$</p> <p>$\frac{1(w+3) + 7(w-6)}{(w-6)(w+3)}$</p> <p>$= \frac{w+3+7w-42}{(w-6)(w+3)}$</p> <p>$= \frac{8w-39}{(w-6)(w+3)}$</p> <p>NPV's: $w \neq 6, -3, -8$</p>	<p>f) $\frac{-4c-4}{c^2+2c-15} + \frac{6c+42}{c^2+12c+35}$</p> <p>$\frac{-4(c+1)}{(c+5)(c-3)} + \frac{6(c+7)}{(c+5)(c+7)}$</p> <p>$\frac{-4(c+1) + 6(c-3)}{(c+5)(c-3)}$</p> <p>$= \frac{-4c-4+6c-18}{(c+5)(c-3)}$</p> <p>$= \frac{2c-22}{(c+5)(c-3)}$</p> <p>NPV's: $c \neq -5, 3, -7$</p>
<p>g) $\frac{a}{a+b} - \frac{b}{b-a} + \frac{2ab}{a^2-b^2}$</p> <p>$\frac{a}{a+b} + \frac{b}{a-b} + \frac{2ab}{(a+b)(a-b)}$</p> <p>$\frac{a(a-b) + b(a+b) + 2ab}{(a+b)(a-b)}$</p> <p>$= \frac{a^2 - ab + ab + b^2 + 2ab}{(a+b)(a-b)}$</p> <p>$= \frac{a^2 + 2ab + b^2}{(a+b)(a-b)} = \frac{(a+b)(a+b)}{(a+b)(a-b)} = \frac{a+b}{a-b}$</p> <p>NPV: $a \neq -b, b$</p>	<p>h) $\frac{2x}{3-x} - \frac{3x}{x+3} + \frac{2}{x^2-9}$</p> <p>$\frac{-2x}{x-3} - \frac{3x}{x+3} + \frac{2}{(x+3)(x-3)}$</p> <p>$\frac{-2x(x+3) - 3x(x-3) + 2}{(x-3)(x+3)}$</p> <p>$= \frac{-2x^2 - 6x - 3x^2 + 9x + 2}{(x-3)(x+3)}$</p> <p>$= \frac{-5x^2 + 3x + 2}{(x-3)(x+3)} = \frac{-(5x+2)(x-1)}{(x-3)(x+3)}$</p>	

NPV's:	NPV's: $x \neq \pm 3 //$
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5. Given each of the following statements, indicate whether if they are true or false: Explain:

i) $\frac{x}{y} - \frac{y}{x} = \frac{x-y}{xy}$ TRUE / FALSE

ii) $\frac{x-y}{y-x} = -1$ TRUE / FALSE

iii) $\frac{wx+wy}{w+v} = \frac{x+y}{v}$ TRUE / FALSE

$= \frac{x^2 - y^2}{xy}$

iv) $\frac{1}{x-y} + \frac{1}{y-x} = 0$ TRUE / FALSE

$\frac{1}{x-y} - \frac{1}{x-y} = 0$

Handwritten notes for problem 5iv:
 $\frac{1}{x-y} + \frac{1}{y-x} = \frac{1}{x-y} - \frac{1}{x-y} = 0$
 $\frac{1}{x-y} + \frac{1}{y-x} = \frac{1}{x-y} - \frac{1}{x-y} = 0$
 $\frac{1}{x-y} + \frac{1}{y-x} = \frac{1}{x-y} - \frac{1}{x-y} = 0$

6. Simplify and find all the NPV's

<p>a) $\frac{x+1}{x+6} + \frac{x^2+4x+3}{x^2+x-6} \times \frac{x^2-2x}{x^2-1}$</p> <p>$\frac{x+1}{x+6} + \frac{(x+3)(x+1)}{(x+3)(x-2)} \times \frac{x(x-2)}{(x+1)(x-1)}$</p> <p>$\frac{x+1}{x+6} + \frac{x}{x-1}$</p> <p>$= \frac{(x+1)(x-1) + x(x+6)}{(x+6)(x-1)}$</p> <p>$= \frac{x^2-1 + x^2+6x}{(x+6)(x-1)} = \frac{6x-1}{(x+6)(x-1)}$</p> <p>NPV's: $x \neq -6, -3, 2, \pm 1$</p>	<p>b) $\frac{x+8}{x-3} - \frac{6x^2+x-2}{24x^2+7x-6} \div \frac{2x^2-7x+3}{8x^2-3x}$</p> <p>$\frac{x+8}{x-3} - \frac{6x^2+x-2}{(8x-3)(3x+2)} \times \frac{(8x-3)}{(2x-1)(x-3)}$</p> <p>$\frac{x+8}{x-3} - \frac{(3x+2)(2x-1)}{(8x-3)(3x+2)} \cdot \frac{(8x-3)}{(2x-1)(x-3)}$</p> <p>$\frac{x+8}{x-3} - \frac{x}{x-3}$</p> <p>$= \frac{8}{x-3}$</p> <p>NPV's: $x \neq 3, \frac{3}{8}, \frac{1}{3}, 0, \frac{1}{2}$</p>
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7. Simplify and state all the NPV's:

$\frac{x-2}{2x} + \frac{1}{x+2} : \frac{3}{2} - \frac{6}{x^2+3x}$

$= \left[\frac{(x-2)}{2x} + \frac{1}{x+2} \right] \cdot \frac{(2x)(x+2)(x+3)}{(2x)(x+2)(x+3)}$ LCD: $(2x)(x+2)(x+3)$

$= \frac{(x-2)(x+2)(x+3) + 2x(x+3)}{3(x+2)(x+3) - 6(2)(x+2)}$

$= \frac{(x+3)(x^2+2x-4)}{(x+2)(3x^2+9x-12)}$

$= \frac{(x+3)(x^2+2x-4)}{(x+2)(3)(x+4)(x-1)}$

NPV: $x \neq 0, -2, -3, -4, 1$

8. Challenge: if $\frac{1}{a(b+1)} + \frac{1}{b(a+1)} = \frac{1}{(a+1)(b+1)}$, then what is the value of $\frac{1}{a} + \frac{1}{b}$?

$\frac{b(a+1) + a(b+1)}{ab(b+1)(a+1)} = \frac{1}{(a+1)(b+1)}$

$= \frac{ab+ab+a+b}{ab(b+1)(a+1)} = \frac{1}{(a+1)(b+1)}$

$\frac{2ab+a+b}{ab} = 1$

$\frac{2ab}{ab} + \frac{a}{ab} + \frac{b}{ab} = 1$

$2 + \frac{1}{b} + \frac{1}{a} = 1$

$\frac{1}{a} + \frac{1}{b} = -1 //$